First Insights in Horizontal Magnetic Field Transfer Functions from 2D and 3D Simulations

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Introduction

The conventional magnetotelluric (MT) transfer functions (impedance tensor and tipper vector) provide information about the Earth´s interior structure. Other transfer functions (e.g. Schmucker & Weidelt [1975], Caldwell [2004], Hering et al. [2019]) deliver further information and are sensitive to different features. However, the effects of subsurface structures on the magnetic field including primary and secondary horizontal magnetic field have been of minor interest so far.

We introduce arbitrary inter-station transfer functions between horizontal magnetic field components. Differences in horizontal magnetic field components can be caused by lateral resistivity contrasts but contrary to the Geomagnetic Depth Sounding perturbation matrix, no reference site in a 1D environment is necessary. This facilitates field data acquisition. Additionally the horizontal magnetic field transfer functions (HMTF) might be able to replace tipper information for sites where no vertical magnetic field data are available.

In this study we show the behavior of the new transfer functions for 2D and 3D environments.

Theory

We calculate the HMTF tensor **T** between two stations a and b according to equation 1.

$$\begin{pmatrix} T_{xx}(f) & T_{xy}(f) \\ T_{yx}(f) & T_{yy}(f) \end{pmatrix} = \begin{pmatrix} Bx_1^a(f) & Bx_2^a(f) \\ By_1^a(f) & By_2^a(f) \end{pmatrix} \cdot \begin{pmatrix} Bx_1^b(f) & Bx_2^b(f) \\ By_1^b(f) & By_2^b(f) \end{pmatrix}^{-1} \quad (1)$$

Subscripts 1 and 2 refer to the two orthogonal polarizations. The phase is calculated as introduced by Caldwell [2004] for the impedance tensor (eq. 2).

$$\begin{pmatrix} \phi_{xx} & \phi_{xy} \\ \phi_{yx} & \phi_{yy} \end{pmatrix} = \Re(\bar{\bar{T}})^{-1} \cdot \Im(\bar{\bar{T}}) \quad (2)$$

We plot the invariants of **ΔT** and **Φ** using a modified version of the ellipse plot introduced by Hering et al. [2019] (s. fig. 1). **ΔT** is a modified matrix reflecting the scattered part of |**T**|.

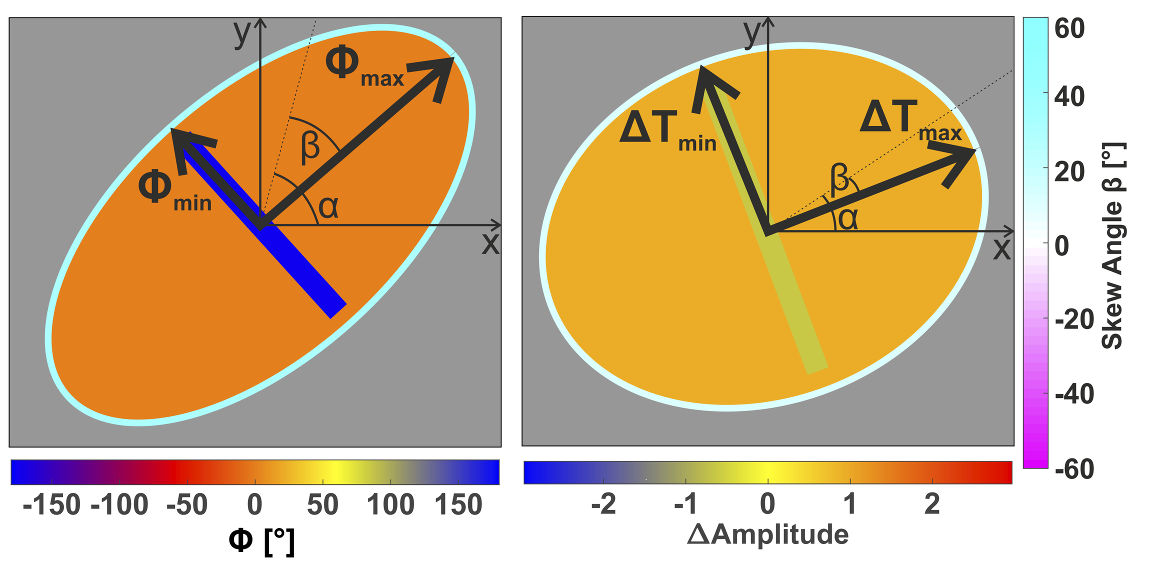

*Fig. 1: The ellipse background colour shows the highest delta amplitude of the HMTF tensor compared to a 1D case (right side). The colour of the bar along the minor axis gives the minimum scattered part ΔT_{min}. The colour of the rim indicates the asymmetry of |**T**| (skew angle β) and the angle α represents its rotation. Left: The invariants of the HMTF phase tensor **Φ** are plotted. If phases vary between -180 and 180 degrees, a continuous colour map is used.*

2D Environment

For a two-dimensional environment only currents flowing parallel to the strike direction produce a secondary magnetic field at the surface (TE mode). When measuring along strike direction, T_{yy} representing the TM mode becomes one while off-diagonal elements are zero (eq. 3 and yellow colour in fig. 3, 4).

$$\bar{\bar{T}} = \begin{pmatrix} 1+\Delta & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \Delta T = \begin{pmatrix} \Delta T_{xx} & \Delta T_{xy} \\ \Delta T_{yx} & \Delta T_{yy} \end{pmatrix} \quad (3)$$

We calculated two 2D models with different conductivities using COMSOL Multiphysics (s. fig. 2).

In figures 3 and 4 the ellipses are arranged similar to geoelectrics pseudo sections with transfer functions replacing the resistivity values. The ellipse set up represents the transfer functions between two related sites, respectively. The vertical direction corresponds to ΔT_{yy} and thus is zero in this particular case.

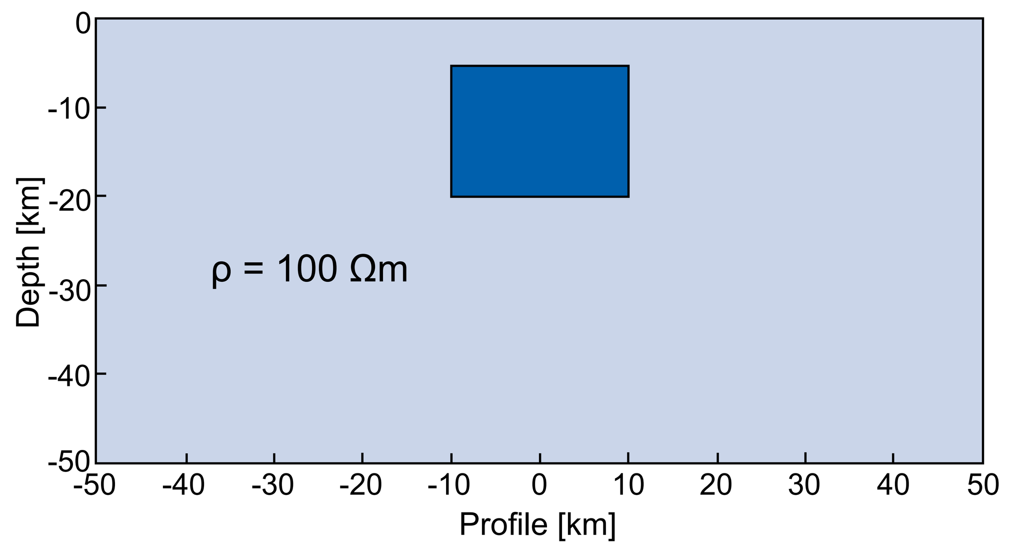

Fig. 2: Set up of the 2D model with homogeneous background resistivity and a 15 km thick anomaly at 5 km depth. The anomaly was set to 1 Ωm and 10.000 Ωm, respectively, and the magnetic field was calculated at 16 stations with equal spacing along a 100 km profile.

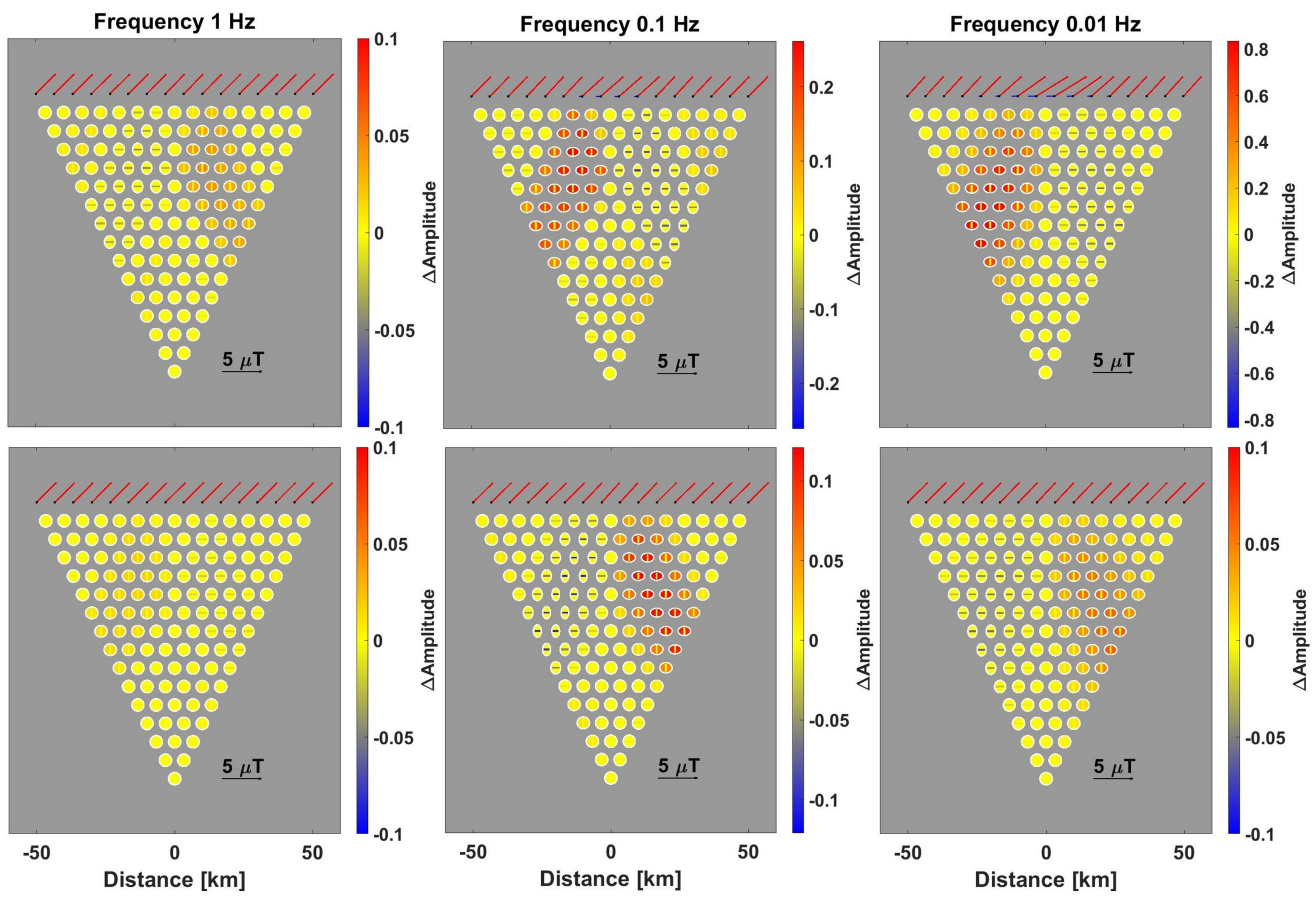

*Fig. 3: Scattered part of |**T**| in a 2D environment with conductive (top) and resistive (bottom) anomaly. ΔT_{xx} (eq. 3) is oriented horizontally and ΔT_{yy} is oriented upwards (here equal zero). The ellipse reflects the orientation of the scattered field (the colour of the ellipse corresponds to the value of its major axis and the colour of the bar represents that of the minor axis). The arrows at the top show the magnetic field at the sites along the profile. The red and blue colour represent the real and imaginary part, respectively. For 2D structures the skew angle is always zero (white rim).*

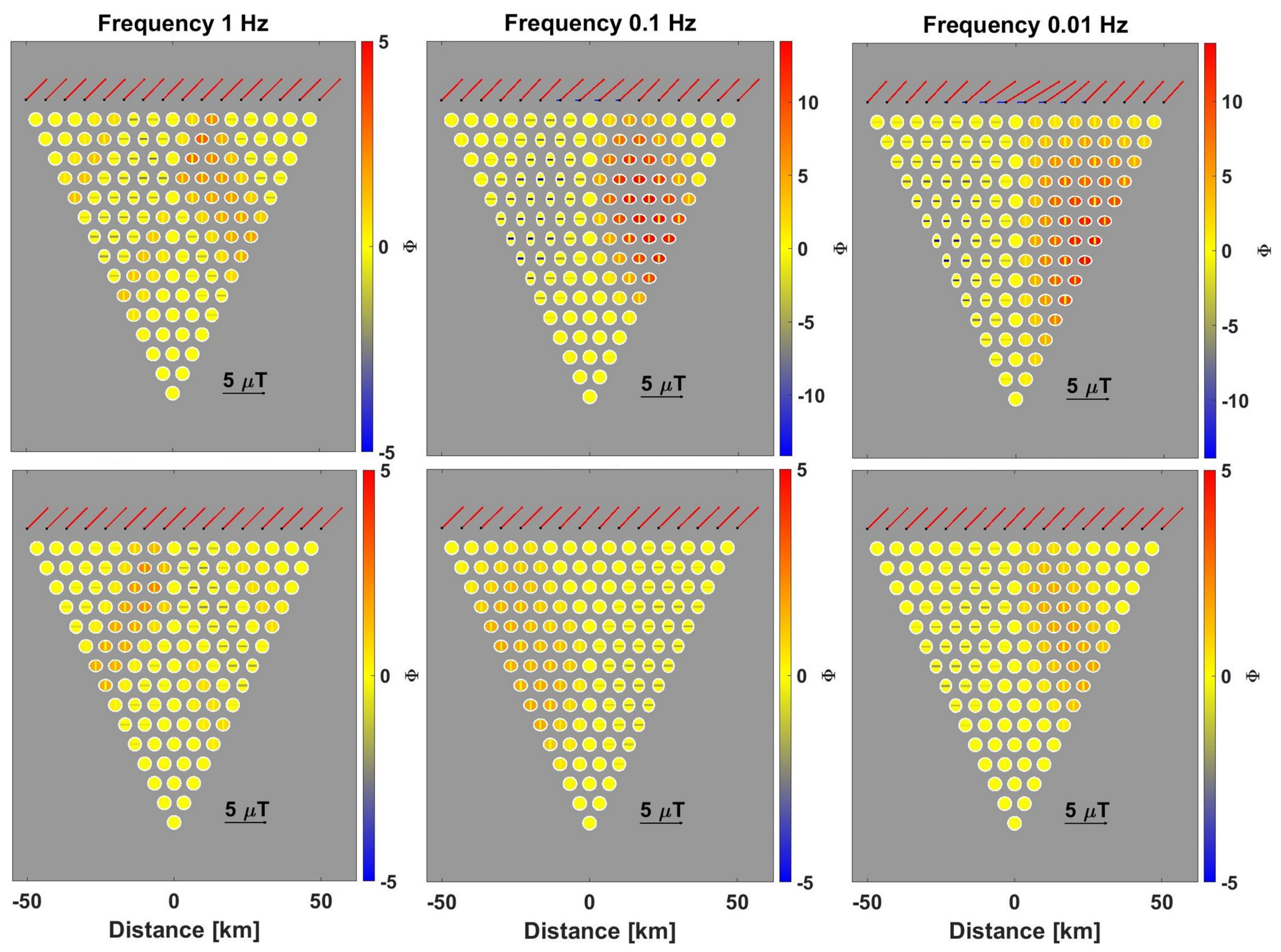

*Fig. 4: Phase of the HMTF tensor **T** in a 2D environment with conductive (top) and resistive (bottom) anomaly.*

3D Environment

For testing HMTFs for a three-dimensional environment we modelled a homogeneous background with 500 Ωm and a 3D anomaly at 1km depth (s. fig. 5). The corner of the anomaly lies in the center of the model and the anomalous resistivity is 1 Ωm.

We show pseudo sections along a profile (red line) and HMTF ellipses in the xy-plane for a fixed reference station (black dot). A clear indicator for 3D-structures is the skew angle β which represents the asymmetry of **T** and thus influences the rotation of the ellipses.

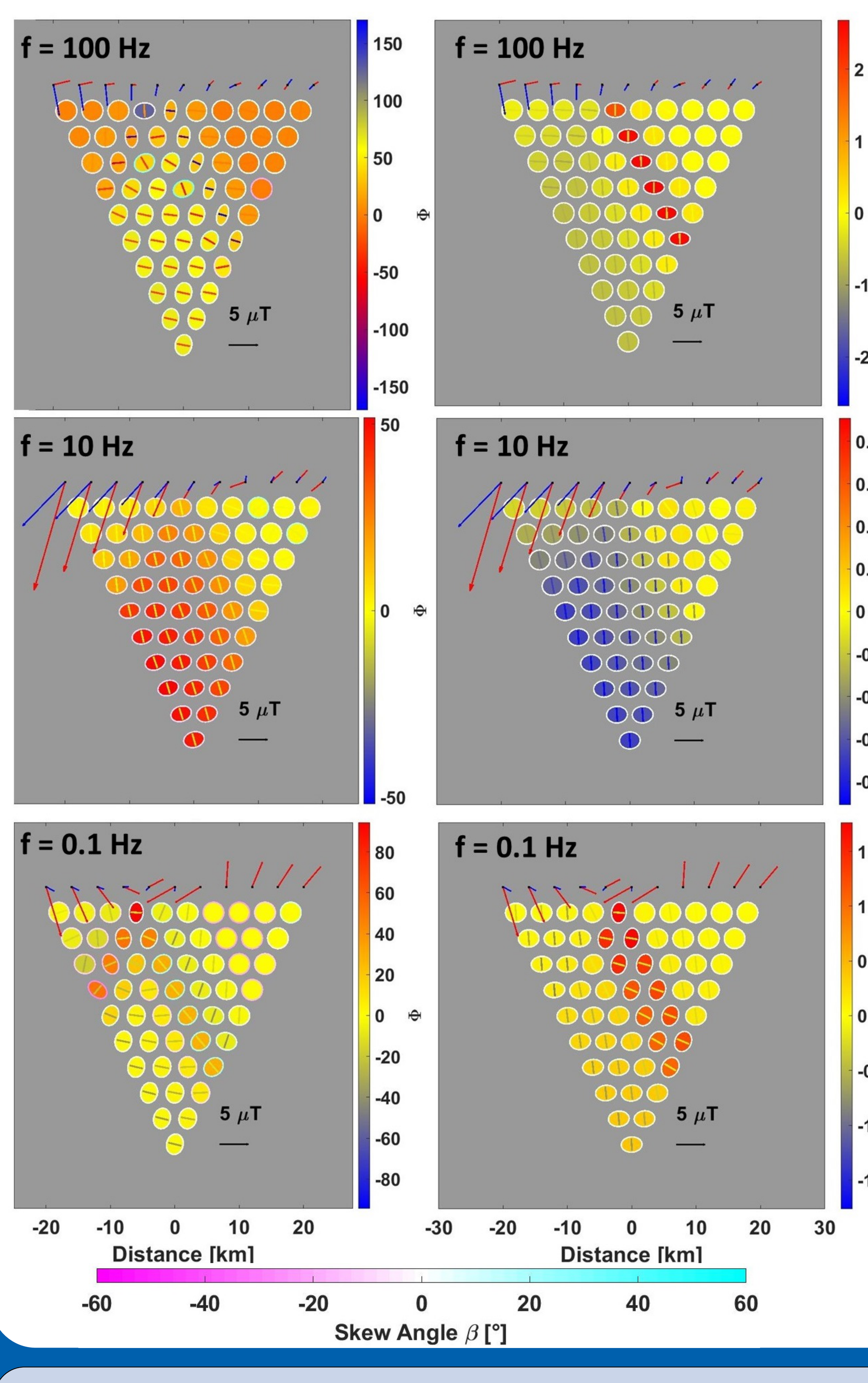

*Fig. 6 (left side): Pseudo sections along a profile in a 3D environment. The scattered field of |**T**| (right column) and phase of the HMTF tensor (left column) are shown for different frequencies.*

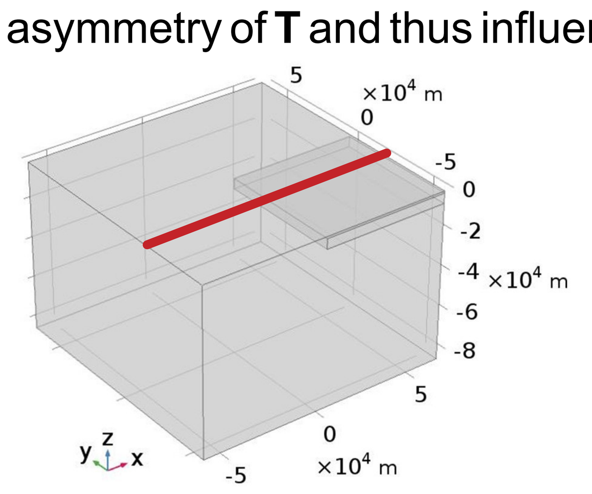

Fig. 5: Set up of the 3D model with a 5 km thick anomaly. The red line marks the profile used for calculating the pseudo sections in figure 6.

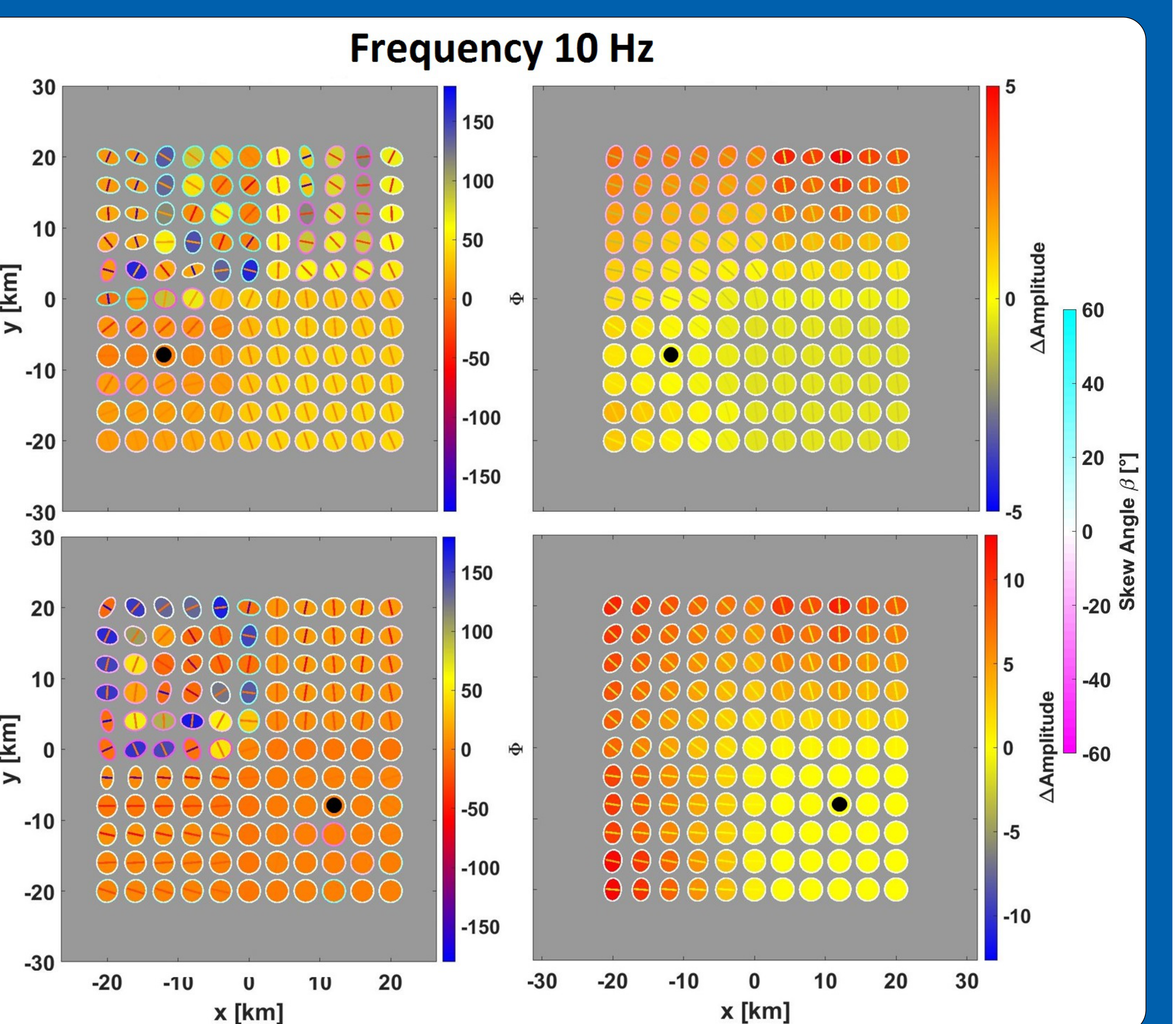

*Fig. 7 (right side): HMTF tensors in a 3D environment plotted in the xy-plane. The scattered field of |**T**| (right column) and phase of the HMTF tensor (left column) are shown for two different reference sites (black dot).*

References:

Schmucker, U. and Weidelt, P., 1975. Electromagnetic induction in the Earth. Lecture notes Aarhus.

Caldwell, T. G., Bibby, H. M., and Brown, C. (2004). The magnetotelluric phase tensor. Geophysical Journal International, 158(2): 457-469.

Hering, Ph., Brown C., Junge A. (2019), Magnetotelluric Apparent Resistivity Tensors for improved Interpretations and 3-D Inversions, Journal of Geophysical Research: Solid Earth, 124, doi: 10.1029/2018JB017221

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